MODEL OF TUBES OF FORCE OF THE EARTH'S MAGNETIC FIELD

The motion considered is that of an electron moving in a region of converging (or diverging) magnetic field lines. The forces acting on the electron are shown in Fig. 1 for the simplest case of a cone of magnetic lines. The semi-angle of the cone is *a*.

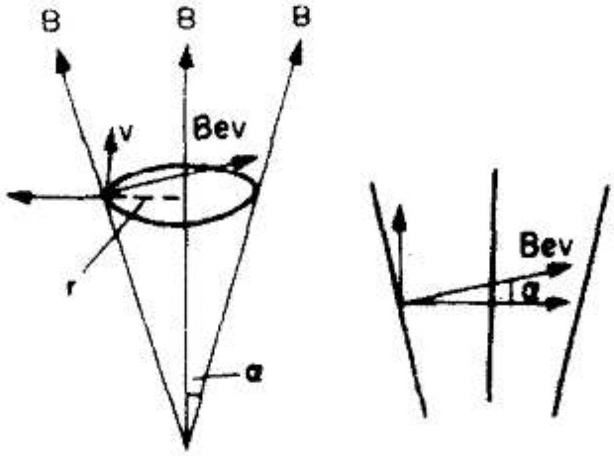

Fig. 1. Electron in converging *B* lines.

In general, the force on a charge moving in a magnetic field (Lorentz force) is

$$F = \gamma\ q\,(V \times B) \qquad (1)$$

where

γ is the electromagnetic constant,

q the charge, and

V its velocity in the field B.

In the present case, the absolute value of this force is

$$|F| = Bev,$$

where *e* is the electronic charge.

Its direction is perpendicular to the field line which describes the surface of the cone, *v* is now the *tangential* velocity of an electron describing the circular orbit, radius *r* and some right section of the cone. In general, the electron will also have a longitudinal component of velocity *u*, in the direction of the guiding centre (or central magnetic field line). This will arise both from the component velocity of the electron in this direction when initially injected into the field, and also as a result of acceleration under the action of the longitudinal component of the Lorentz force.

For an electron spiralling in the Earth's field, a first approximation for the configuration of the field lines confining the electron and defining its trajectory could be two such cones as those above, one for motion in each hemisphere. This cone approximation is retained in the following *only* in expressions for the Larmor radius at various points along the field line. However, in the actual case of the magnetosphere, the relative directions of the force *Bev* change as the electron moves along the line of force. That is, a (the angle between this direction and the perpendicular to the guiding centre) is *not* a constant angle (unlike Fig. 1). It will in fact increase from zero at the equatorial plane to a maximum value at the mirroring point (or at the 'surface' if mirroring does not occur). For this reason the assumed model for tubes of force of the magnetic field which is used in determining components of the Lorentz force is that shown in Fig. 2. It is now supposed that the sides of the longitudinal

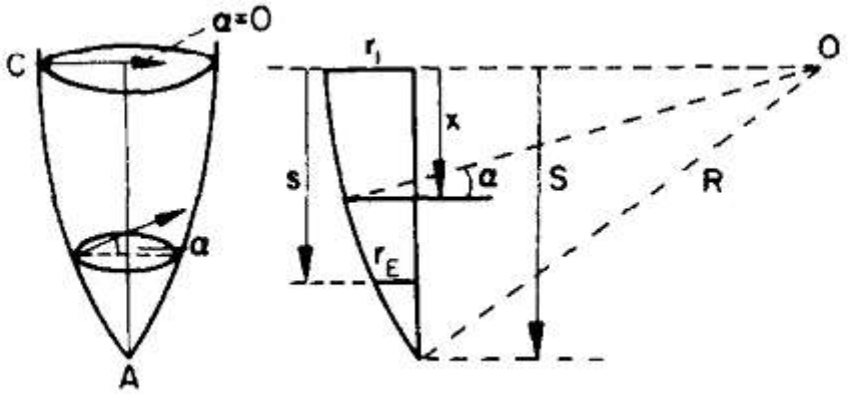

Fig. 2. Model of tubes of force of the Earth's magnetic field.

section of the tube of force are not straight lines (as for the cone) but are arcs of a circle, centre *O*, radius *R*. The direction of the line of force *Bev* (perpendicular to *v* and to *B*—or towards *O* in the section) changes from *A* to *C*. r_1 and r_E are, respectively, the Larmor radii at the equatorial plane and (ideally) at the Earth's surface. The distance along a given field line to a particular point is *x*; to the surface, *s*; and to the (extrapolated) end of the tube, *S*. Such a model also involves, of course, the straightening out of the tube of force actually curved around the Earth's surface.

What is required from this model is a value for *sin a* which will appear in expressions for the longitudinal component of force.

$$\sin a = \frac{2xr_1}{S^2 + r_1^2} \cong \frac{2xr_1}{S^2} \text{ (as } r_1 \ll S)$$

EARTH'S MAGNETIC FIELD

For the purposes of calculation, it is supposed that the Earth is a perfect sphere and that its magnetic field is that of a dipole of moment 1035 * 10^{17} weber-metres. A particular point on a field line is designated by the polar coordinates (*r*, *0*), with θ_0 measured from the direction of the magnetic moment (i.e. co-latitude). A given field line is specified by θ_0, the co-latitude of its point of intersection with the Earth's surface. The constant for the equation of a line of force ($sin^2\ \theta_0/r$) is *k*.

In integrating the work done by the longitudinal force on the spiralling electron, an expression is needed for the increment *dx* in path length along the field line. From the dipole field geometry it can be shown that,

$$dx = \frac{\sin\theta}{k}(3\cos^2\theta + 1)d\theta$$

In the equations of motion of the following sections, values are also required for *x/s* and *x/S* (as defined in Fig. 2) as functions of 0. *x/s* is dependent on θ_0 as well. These, and averaged values of *x* over 5° intervals, can be calculated and tabulated.